HIGH SCHOOL MATHEMATICS CONTEST
Sponsored by
THE MATHEMATICS DEPARTMENT
of
WESTERN CAROLINA UNIVERSITY

LEVEL I
March 17, 2016

Prepared by:

Nathan Borchelt   Andrew Chockla   Axelle Faughn

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each of the 30 problems there are listed up to 5 possible answers. You are to work each problem and determine which is the correct answer. Indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. Here is a sample question and answer:

1. If $2x = 3$, then x equals:

    (A) $\frac{2}{3}$ (B) 3   (C) 6   (D) $\frac{3}{2}$ (E) None of the answers (A) through (D) is correct.

The correct answer for the sample is "$\frac{3}{2}$," which is answer (D); therefore, you should answer this question by making a heavy black mark under space D as indicated below.

A   B   C   D   E
○   ○   ○   ●   ○

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any question. If you are unable to work any particular problem, it is to your advantage to guess at the answer rather than leave it blank. Make no stray marks of any kind on your answer sheet.

When told to do so, open your test booklet to page 2 and begin work. When you have finished one page, go on to the next page. The working time for the entire test is 70 minutes.

The use of calculators is not permitted.

1. Which of the following polynomial functions contain the zeros $-\frac{2}{3}$ and 4?

   (A) $f(x) = 3x^2 + 10x + 4x - 8$ (B) $f(x) = 3x^2 + 14x + 8$ (C) $f(x) = 3x^2 - 10x - 8$
   (D) $f(x) = 3x^2 - 14x + 8$ (E) None of the answers (A) through (D) is correct.

2. What is the least common multiple of $12a^2bc^3$ and $15ab^4c^2$?

   (A) $60a^3b^5c^5$ (B) $180a^2b^4c^6$ (C) $180a^2b^4c^3$ (D) $60a^2b^4c^3$
   (E) None of the answers (A) through (D) is correct.

3. The function $g(x)$ is a straight line with $g(2) = 5$ and $g(-3) = 4$. Find the equation of a line that passes through the point (10,6) and is parallel to $g(x)$.

   (A) $y = \frac{1}{5}x + 4$ (B) $y = \frac{1}{5}x - 8$ (C) $y = 5x - 44$ (D) $y = 5x - 56$
   (E) None of the answers (A) through (D) is correct.

4. The product of two negative, consecutive, odd integers is 63, what is the sum of the integers.

   (A) -2 (B) -12 (C) 2   (D) 16   (E) None of the answers (A) through (D) is correct.

5. Suppose $f(x) = e^x$ and the graph of $g(x)$, shown below, displays a transformation of $f(x)$.
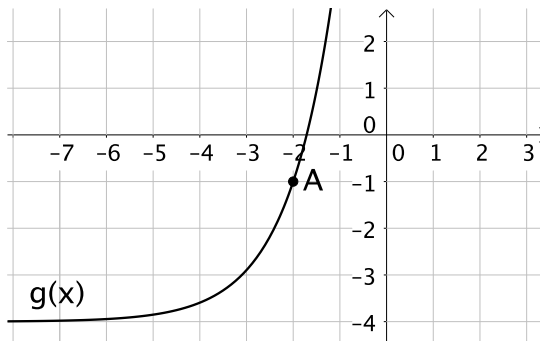
   Assume that point A, displayed on the graph, was originally the y-intercept of $f(x)$. Which of the following is an appropriate way to write $g(x)$ in terms of $f(x)$?

   (A) $g(x) = 3 \cdot f(x - 2) - 4$ (B) $g(x) = 3 \cdot f(x + 2) - 1$ (C) $g(x) = 3 \cdot f(x + 2) - 4$
   (D) $g(x) = 3 \cdot f(x - 2) - 1$ (E) None of the answers (A) through (D) is correct.

6. The following graph is a scatterplot that shows the relationship between x, years since 1989, and y, number of organ transplants performed in the United States. The line displayed on the plot is the line of best fit.

$$\hat{y} = 14.213 + 0.79x$$
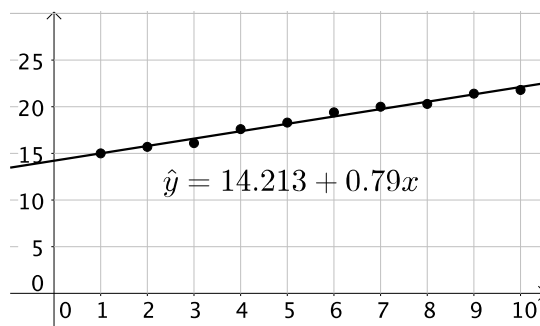
Which of the following describes the relationship between x and y?

(A) Not linear   (B) Strong, negative   (C) Weak, positive   (D) Weak, negative

(E) None of the answers (A) through (D) is correct.

7. If $\dfrac{x}{y} = \dfrac{3}{2}$ and $\dfrac{y}{z} = \dfrac{2}{7}$, then what is the value of $\dfrac{x}{z}$?

(A) $\frac{4}{21}$ (B) $\frac{7}{4}$ (C) $\frac{5}{14}$ (D) $\frac{3}{7}$ (E) None of the answers (A) through (D) is correct.

8. A recipe calls for $\frac{2}{3}$ of a cup of sugar. You find that you have only $\frac{1}{2}$ cup of sugar left. What fraction of the recipe can you make?

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{3}{2}$ (D) $\frac{3}{4}$ (E) None of the answers (A) through (D) is correct.

9. Which of the following describes the graph of $x + y = 3$ in the x-y plane?

(A) A line with positive slope   (B) A line with negative slope   (C) A horizontal line
(D) A pair of intersecting lines   (E) None of the answers (A) through (D) is correct.

10. The average of five numbers is 30. After one of the numbers is removed, the average arithmetic mean of the remaining numbers is 32. What number was removed?

(A) 17   (B) 22   (C) 15   (D) 25   (E) None of the answers (A) through (D) is correct.

11. The midpoint of the segment containing (6,10) and (20,8) is (x,y). What is the product of x and y?

(A) 143   (B) 132   (C) 128   (D) 117   (E) None of the answers (A) through (D) is correct.

12. If $c \clubsuit d = 2c + d - 1$, what is the value of $(3 \clubsuit 10) \clubsuit 6$?

(A) 36   (B) 23   (C) 24   (D) 35   (E) None of the answers (A) through (D) is correct.

3

13. What is the number of hours from 5pm Thursday until 9am Saturday of the same week?

(A) 40   (B) 38   (C) 41   (D) 36   (E) None of the answers (A) through (D) is correct.

14. For what value of t does $\dfrac{3^{4t-1}}{3^{5t+6}} = (9^{2t-1})(3^{-t+4})$?

(A) $\frac{3}{2}$ (B) $\frac{-9}{4}$ (C) $\frac{-9}{2}$ (D) $\frac{3}{4}$ (E) None of the answers (A) through (D) is correct.

15. Find the perimeter of a triangle which has vertices at (-2,3), (-2,-3),and (6,-3).

(A) 32   (B) 24   (C) $20 + 2\sqrt{13}$ (D) $14 + 2\sqrt{7}$
(E) None of the answers (A) through (D) is correct.

16. Consider the following graph of lines $\ell_1 = \frac{-1}{2}x + 2$ and $\ell_2 = \frac{3}{4}x - 3$.
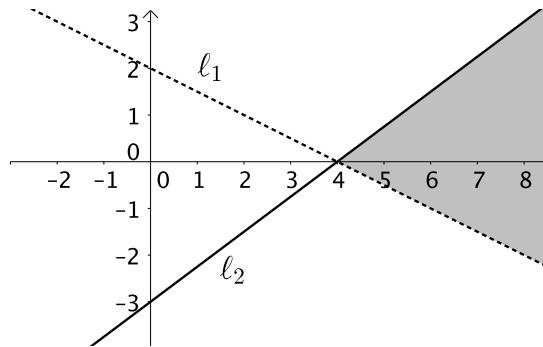

Which of the following set of inequalities represents the shaded region of the graph above?

(A) $y \leq \frac{3}{4}x - 3$ and $y > \frac{-1}{2}x + 2$ (B) $y < \frac{3}{4}x - 3$ and $y \geq \frac{-1}{2}x + 2$ (C) $y \geq \frac{3}{4}x - 3$ and $y < \frac{-1}{2}x + 2$
(D) $y > \frac{3}{4}x - 3$ and $y \leq \frac{-1}{2}x + 2$ (E) None of the answers (A) through (D) is correct.

17. Abel is putting triangles together using toothpicks. If he follows the pattern below, how many toothpicks will it take to make 50 attached triangles?

1 triangle
3 toothpicks

2 triangles
5 toothpicks

3 triangles
7 toothpicks
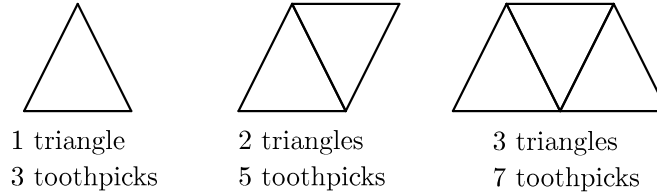
(A) 100   (B) 51   (C) 97   (D) 101   (E) None of the answers (A) through (D) is correct.

18. Three or more points are said to be collinear if they lie on a single straight line. If the points (1,3), (5,5), and $(4r, r)$ are collinear, what is the value of r?

(A) $\dfrac{-5}{2}$ (B) $\dfrac{-1}{7}$ (C) $\dfrac{7}{5}$ (D) $\dfrac{5}{3}$ (E) None of the answers (A) through (D) is correct.

19. Tickets for the county fair cost $10 for a single person or $16 for two people. A total of $900 was collected and 72 people were admitted to the fair. How many single person tickets were sold?

(A) 36   (B) 40   (C) 42   (D) 60   (E) None of the answers (A) through (D) is correct.

20. How many Triangles are there in this drawing?
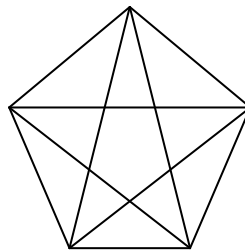
(A) 10   (B) 25   (C) 30   (D) 35   (E) None of the answers (A) through (D) is correct.

21. What is the sum of the solutions to the following equation?

$$2x^2 - 3x = 9 - 3x^2$$

(A) 2   (B) $\frac{3}{5}$ (C) $6\sqrt{21}$ (D) $\frac{-3}{10}$ (E) None of the answers (A) through (D) is correct.

22. A standard coin is tossed four times. What is the probability that heads comes up exactly twice?
    (A) $\frac{5}{8}$ (B) $\frac{7}{16}$ (C) $\frac{3}{8}$ (D) $\frac{1}{2}$ (E) None of the answers (A) through (D) is correct.

23. A survey of 147 college students was done to find out which elective courses they were taking. The study revealed the following information:

45	were taking Art
55	were taking Basket Weaving
40	were taking Canoeing
12	were taking Art and Basket Weaving
15	were taking Art and Canoeing
23	were taking Basket Weaving and Canoeing
2	were taking all three classes

   How many students were not taking any of these three electives?
    (A) 7   (B) 10   (C) 49   (D) 55   (E) None of the answers (A) through (D) is correct.

24. For how many pairs of positive integers (x, y) does $5x + y = 165$?
    (A) 165   (B) 33   (C) 20   (D) 32   (E) None of the answers (A) through (D) is correct.

25. In triangle ABC, segment CE has length 50 units, segment EA has length 25 units, and segment DE has length 10 units. What is the area of triangle ABC?
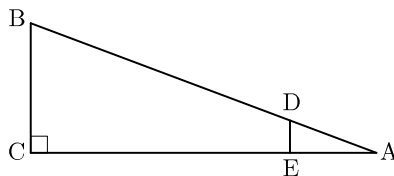
(A) 750 square units   (B) 1125 square units   (C) 1500 square units   (D) 2250 square units
(E) None of the answers (A) through (D) is correct.

26. For what values of k is $4^{|3k-4|} \geq 8$?

    (A) $\left[\frac{2}{3}, 2\right]$ (B) $\left[\frac{5}{6}, \frac{11}{6}\right]$ (C) $\left(-\infty, \frac{2}{3}\right] \cup [2, \infty)$ (D) $\left(-\infty, \frac{5}{6}\right] \cup \left[\frac{11}{6}, \infty\right)$
    (E) None of the answers (A) through (D) is correct.

27. The box plot below represents Big Mac prices in U.S. dollars for 45 different countries.
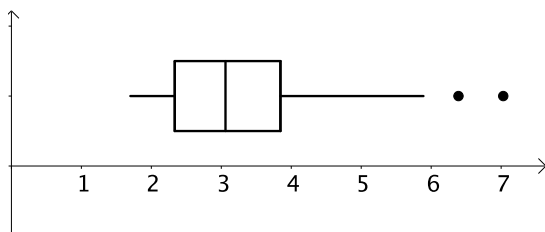
    75% of the prices from the data set are above which value?

    (A) $3.85   (B) $2.34   (C) $3.06   (D) $5.89   (E) None of the answers (A) through (D) is correct.

28. Fred can mow a lawn in 3 hours. Joe can accomplish the same feat in 2 hours. If Fred, Joe, and Susan work together to mow the lawn, they can get it done in $\frac{12}{13}$ of an hour. How long does it take for Susan to mow the lawn herself?

    (A) 2 hours   (B) 3 hours   (C) 4 hours   (D) 5 hours
    (E) None of the answers (A) through (D) is correct.

29. Find the number of distinct integer solutions to the equation $(x^2 - x - 1)^{x+2} = 1$.

    (A) 2   (B) 3   (C) 4   (D) 5   (E) None of the answers (A) through (D) is correct.

30. Given the recursive function defined by

$$f(1) = 1,$$
$$f(2) = 2,$$
$$f(n) = 2f(n-1) + f(n-2) \text{ for } n \geq 3,$$

    what is the value of $f(5)$?

    (A) 5   (B) 12   (C) 29   (D) 70   (E) None of the answers (A) through (D) is correct.