

HIGH SCHOOL MATHEMATICS CONTEST
Sponsored by
THE MATHEMATICS DEPARTMENT
of
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LEVEL II TEST
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DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each of the 30 problems there are listed up to 5 possible answers. You are to work each problem and determine which is the correct answer. Indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. Here is a sample question and answer:

1. If $2x = 3$, then x equals:

- (A) $\frac{2}{3}$ (B) 3 (C) 6 (D) $\frac{3}{2}$ (E) None of the answers (A) through (D) is correct.

The correct answer for the sample is " $\frac{3}{2}$," which is answer (D); therefore, you should answer this question by making a heavy black mark under space D as indicated below.

A B C D E

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any question. If you are unable to work any particular problem, it is to your advantage to guess at the answer rather than leave it blank. Make no stray marks of any kind on your answer sheet.

When told to do so, open your test booklet to page 2 and begin work. When you have finished one page, go on to the next page. The working time for the entire test is 70 minutes.

The use of calculators is not permitted.

1. A parabola defined by $y = x^2 + 11x + 14$ is intersected twice by the line $3x - y = 1$. What is the product of the two x -coordinates of the points of intersection?
 (A) 15 (B) 8 (C) 12 (D) 14 (E) None of the answers (A) through (D) is correct.
2. Solve the equation $2^{x+3} = 4^{x-7}$.
 (A) $x = 10$ (B) $x = 12$ (C) $x = 15$ (D) $x = 17$ (E) None of the answers (A) through (D) is correct.
3. Five squares are lined side-by-side so that the perimeter of the resulting rectangle is 36 inches. What is the area of the rectangle?
 (A) 12 sq in (B) 45 sq in (C) 36 sq in (D) 50 sq in (E) None of the answers (A) through (D) is correct.
4. Suppose a bag contains two black marbles, three red marbles, and four green marbles, all of identical size. Rebecca randomly removes one marble from the bag and keeps it in her hand, followed by Kaylin who randomly removes another marble from the bag and keeps it in her hand. What is the probability that neither girl has a black marble in her hand?
 (A) $1/18$ (B) $7/12$ (C) $14/27$ (D) $8/9$ (E) None of the answers (A) through (D) is correct.
5. Consider the equation of a line that passes through the center of the circle $x^2 + 2x + y^2 - 6y = 1$ and is perpendicular to the line $y = 2x - 3$. This line can be written in the form $y = mx + b$. Which of the following is equal to $\frac{m}{b}$?
 (A) $-2/7$ (B) $-3/7$ (C) $-4/5$ (D) $-1/5$ (E) None of the answers (A) through (D) is correct.
6. For what positive value of k will the graph of the function $y = \frac{5}{16x^2 + kx + 9}$ have exactly one vertical asymptote?
 (A) 0 (B) 12 (C) 24 (D) 48 (E) None of the answers (A) through (D) is correct.
7. If $ABCD$ is a parallelogram where $AB = x + 30$, $BC = 4x - 10$, and $CD = 2x + 10$, find the length of AD .
 (A) 25 (B) 40 (C) 50 (D) 60 (E) None of the answers (A) through (D) is correct.
8. If $2^x = 3$, then 4^{-x} is:
 (A) $1/3$ (B) 3 (C) $1/9$ (D) 9 (E) None of the answers (A) through (D) is correct.

9. Triangle ABC is similar to triangle DEF , with the following measurements:

$$AB = x - 4, \quad AC = x + 2, \quad BC = x, \quad DF = 2x + 4, \quad DE = 2x - 8, \quad EF = 10.$$

Find the value of x .

- (A) 5 (B) 6 (C) 8 (D) 10 (E) None of the answers (A) through (D) is correct.
10. The system of equations

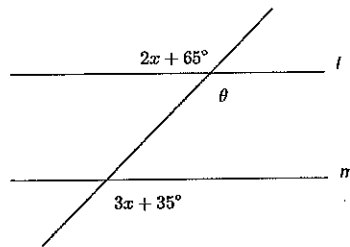
$$\begin{aligned}x + 2y &= 7 \\3x - y + z &= -2 \\2x + 3z &= -4\end{aligned}$$

has a unique solution (x, y, z) . Calculate $2x + 3y - z$.

- (A) 8 (B) 12 (C) 9 (D) 14 (E) None of the answers (A) through (D) is correct.
11. If $f(x)$ is a function with $f(0) = 3$, $f(4) = 2$ and $f(-4) = 1$, what is the y -intercept of the function $g(x) = 2f(x - 4)$?
- (A) -6 (B) 2 (C) 4 (D) -2 (E) None of the answers (A) through (D) is correct.

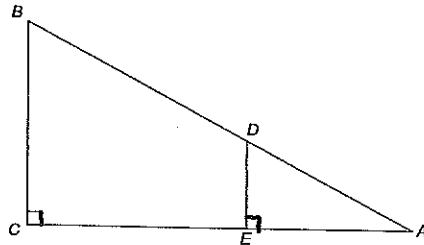
12. What is the exact value of $\frac{\sqrt{15} - \sqrt{7}}{\sqrt{15} + \sqrt{7}} + \frac{\sqrt{15} + \sqrt{7}}{\sqrt{15} - \sqrt{7}}$?
- (A) 11 (B) $11/2$ (C) $7/2$ (D) $3(\sqrt{15} + \sqrt{7})$ (E) None of the answers (A) through (D) is correct.

13. In the figure, lines l and m are parallel. What is the measure of angle θ ?




- (A) 125° (B) 97° (C) 30° (D) 16° (E) None of the answers (A) through (D) is correct.
14. Find the base a for which $\log_a 3 = \sqrt[3]{27}$.
- (A) $\sqrt{3}$ (B) 3 (C) $\sqrt[3]{3}$ (D) $1/3$ (E) None of the answers (A) through (D) is correct.

15. There are two solutions (a, b) of $4^{15x^2-13} = 4^{2x-5}$. Calculate $a \cdot b$.
 (A) $-8/15$ (B) $4/3$ (C) $-5/12$ (D) $6/7$ (E) None of the answers (A) through (D) is correct.
16. Which of the following statements is true regarding the graph of $f(x) = \frac{3x^2 + 3x - 18}{5x^2 - 45}$?
 (A) It has a vertical asymptote at $x = -3$. (B) It has a vertical asymptote at $x = 3$. (C) It has a horizontal asymptote at $y = 3/5$. (D) Exactly two of the above statements (A, B, C) are true. (E) Each of the above statements (A, B, C) are true.
17. Which of the following is equivalent to $(1 + i)^8$?
 (A) 0 (B) $1 + 8i$ (C) $16i$ (D) 16 (E) $16 + 16i$
18. What is the distance between the centers of the circles $x^2 + 2x + y^2 - 6y = 1$ and $x^2 - 8x + y^2 - 4y = 1$?
 (A) $\sqrt{2}$ (B) $\sqrt{21}$ (C) $\sqrt{32}$ (D) $\sqrt{43}$ (E) None of the answers (A) through (D) is correct.
19. In triangle ABC , segment CE has length 50 units, segment EA has length 25 units, and segment DE has length 10 units. What is the area of triangle ABC ?



- (A) 2250 square units (B) 1500 square units (C) 1125 square units (D) 750 square units
 (E) None of the answers (A) through (D) is correct.
20. Which of the following is equivalent to $\frac{1-x}{1-\frac{1}{1-\frac{1}{x}}}$?
 (A) $(1-x)^2$ (B) $x-1$ (C) $1-x$ (D) $1+x$ (E) 1
21. Let $f(x) = \frac{x+1}{x+5}$. What is the simplified form of $(f \circ f)(x)$?
 (A) $\left(\frac{x+1}{x+5}\right)^2$ (B) $\frac{x+3}{3x+13}$ (C) $\frac{3}{2x+13}$ (D) $\frac{x+1}{x+6}$ (E) 5

22. The logarithm $\log\left(\frac{100(x-5)^2x^3}{\sqrt[4]{x+2}}\right)$ is equivalent to which of the following:
 (A) $2 + 2\log(x-5) + 3\log(x) - \frac{1}{4}\log(x+2)$ (B) $2 + \frac{19}{4}\log(x) - 2\log(5) - \frac{1}{4}\log(2)$
 (C) $100 + \log(x^2 - 25) + 3\log(x) - \log(x^{1/4} + \frac{1}{2})$ (D) $100 + \frac{19}{4}\log(x) - \frac{9}{4}\log(3)$ (E) None of the answers (A) through (D) is correct.
23. A triangle ABC has $\angle A = 30^\circ$, $\angle C = 45^\circ$, $AB = x$, and $BC = 4$. Find x .
 (A) $4\sqrt{2}$ (B) $\sqrt{6}$ (C) $\sqrt{2}$ (D) $2\sqrt{6}$ (E) None of the answers (A) through (D) is correct.
24. Suppose the graph of a function $y = f(x)$ is a parabola with vertex $(2, 3)$. Where is the vertex of the parabola given by $y = f(5x - 1)$?
 (A) $(2/5, 2)$ (B) $(10, 2)$ (C) $(5/2, 3)$ (D) $(3/5, 3)$ (E) None of the answers (A) through (D) is correct.
25. For which values of x is it true that $\ln(x-1) + \ln(x+1) = \ln(2)$?
 (A) $x = -1$ and $x = 1$ (B) $x = 1$ only (C) $x = -\sqrt{3}$ and $x = \sqrt{3}$ (D) $x = \sqrt{3}$ only
 (E) None of the answers (A) through (D) is correct.
26. Suppose A and B are independent events so that the probability of A is 0.2 and the probability of B is 0.4. What is the probability of A or B (or both) occurring?
 (A) 0.08 (B) 0.2 (C) 0.52 (D) 0.6 (E) 0.82
27. If $x - 4$ divides the polynomial $x^4 + rx^3 + rx^2 + rx - 4$ with a remainder of 0, what is the value of r ?
 (A) 3 (B) 1 (C) 0 (D) -1 (E) -3
28. What are the real numbers that satisfy the inequality $|x^3 - 9x| - |x^2 - 9| \geq 0$?
 (A) $(-\infty, \infty)$ (B) $(\infty, -3] \cup [3, \infty)$ (C) $(\infty, -1] \cup [1, \infty)$ (D) $[-1, 1]$ (E) $[-3, 3]$
29.  O M IT.
30. Suppose z varies jointly with $\frac{1}{x+1}$ and y^2 , and $z = 16$ when $x = 1$ and $y = 2$. What is z when $x = 3$ and $y = 4$?
 (A) 32 (B) 8 (C) 64 (D) 16 (E) None of the answers (A) through (D) is correct.