HIGH SCHOOL MATHEMATICS CONTEST
Sponsored by
THE MATHEMATICS DEPARTMENT
of
WESTERN CAROLINA UNIVERSITY

LEVEL I TEST
March 23, 2017

Prepared by:

John Wagaman, Chairperson   Nathan Borchelt

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each of the 30 problems there are listed up to 5 possible answers. You are to work each problem and determine which is the correct answer. Indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. Here is a sample question and answer:

1. If $2x = 3$, then x equals:

    (A) $\frac{2}{3}$ (B) 3   (C) 6   (D) $\frac{3}{2}$ (E) None of the answers (A) through (D) is correct.

The correct answer for the sample is "$\frac{3}{2}$," which is answer (D); therefore, you should answer this question by making a heavy black mark under space D as indicated below.

A   B   C   D   E
○   ○   ○   ●   ○

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any question. If you are unable to work any particular problem, it is to your advantage to guess at the answer rather than leave it blank. Make no stray marks of any kind on your answer sheet.

When told to do so, open your test booklet to page 2 and begin work. When you have finished one page, go on to the next page. The working time for the entire test is 70 minutes.

The use of calculators is not permitted.

1

1. The mean of 5 numbers is 13. Which of the following must be true?

   (A) The middle number is 13.   (B) Each number is 13.   (C) The difference between the smallest number and the largest number is 13.   (D) The sum of the numbers is 65.   (E) The product of the numbers is 65.

2. Find the sum of the factors of $3x^2 + 2x - 5$.

   (A) $4x + 4$ (B) $4x - 4$ (C) $4x - 6$ (D) $4x + 6$ (E) $4x - 5$

3. If the graph of a line has a positive slope and a negative y-intercept, what happens to the x-intercept if the slope and the y-intercept are doubled?

   (A) The x-intercept becomes four times larger.   (B) The x-intercept becomes twice as large.   (C) The x-intercept becomes one-fourth as large.   (D) The x-intercept remains the same.   (E) The x-intercept becomes one-half as large.

4. Which of the following equations does not have the same solution as the others?

   (A) $2p - 7 = 13 - 8p$ (B) $\frac{7}{p+5} = \frac{4}{2p}$ (C) $2^p - p^2 = 1$ (D) $(p+2)^2 = p^2 + 12$ (E) All of the choices (A) through (D) have the same solution.

5. The pages of a book are numbered consecutively, starting with page 1. It takes 258 digits to number all of the pages. What is the last page number?

   (A) 122   (B) 104   (C) 108   (D) 116   (E) None of the answers (A) through (D) is correct.

6. What is the simplified form of $\left(\frac{-5a}{3}\right)^{-3}$?

   (A) $\frac{27}{125a^3}$ (B) $-\frac{27}{125a^3}$ (C) $\frac{125a^3}{27}$ (D) $-\frac{125a^3}{27}$ (E) $\frac{15a^3}{9}$

7. What is the greatest common factor of $216x^3y^2z$ and $144x^2y^3$?

   (A) $36x^3y^2z$ (B) $72xyz$ (C) $72x^2y^2$ (D) $432x^3y^3z$ (E) None of the answers (A) through (D) is correct.

8. Find the sum of all values of x such that $3^{|2x+1|} = 27$.

   (A) -1   (B) 0   (C) 1   (D) 2   (E) None of the answers (A) through (D) is correct.

9. If a, b and c are positive integers and $a \clubsuit b \diamondsuit c = \frac{b+c}{a} - \frac{a+c}{b} - \frac{a+b}{c}$, then find $2 \clubsuit 4 \diamondsuit 6$.

   (A) 2   (B) 3   (C) 4   (D) 6   (E) 8

10. if $(x + y)^2 = 18$ and $xy = 6$, what is $x^2 + y^2$?

    (A) 12   (B) $\sqrt{18}$ (C) 6   (D) $\sqrt{6}$ (E) None of the answers (A) through (D) is correct.

11. How many integral values of x satisfy the inequality: $x - 4 \le 5x + 3 \le x + 30$?

    (A) 5   (B) 6   (C) 7   (D) 8   (E) 9

12. If 4 students can sell 15 cookies in 12 minutes, how many minutes will it take for 6 students to sell 45 cookies (assuming they all sell cookies at the same rate)?

    (A) 24   (B) 20   (C) 54   (D) 46   (E) 32

13. If each circle passes through the center of the other circle, and each circle has a circumference of 18π, what is the perimeter of the rectangle?
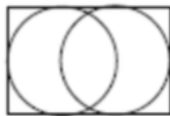

(A) 45   (B) 60   (C) 90   (D) 243   (E) 486

14. A fruit grower packs apples in boxes with each box containing at least 240 apples and at most 250 apples. How many boxes of apples must be chosen to be certain that at least three boxes contain the same number of apples?

(A) 23   (B) 30   (C) 13   (D) 21   (E) 33.

15. How many squares are in the following figure?
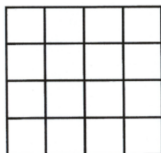
(A) 16   (B) 17   (C) 29   (D) 30   (E) 31

16. If $-1 < x < 0$, then $|x - \frac{1}{x}| + |\frac{1}{x} + x| =$

(A) $-2x$ (B) $2x$ (C) 0 (D) $-\frac{2}{x}$ (E) $\frac{2}{x}$

17. Given $f(x) = \frac{2x-3}{x+1}$, solve for a where $f(3a - 2) = 1$.

(A) 2   (B) 4   (C) 1/2   (D) 1   (E) None of the answers (A) through (D) is correct.

18. Find the y-intercept of the line l shown in the following figure.
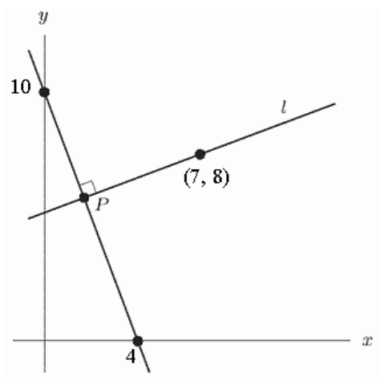
(A) 5   (B) 21/4   (C) 26/5   (D) 6   (E) None of the answers (A) through (D) is correct.

19. Recently, the WCU Math Club hosted a huge Pi Day Celebration at which anyone could come and enjoy a free slice of pie. The choices were apple pie, pecan pie or chocolate cream pie. They ordered an equal number of pecan and chocolate cream pies. They ordered 2 more apple pies than they did pecan pies. The total number of pies ordered was 17. Apple pies cost 9 dollars each while the other pies cost 8 dollars each. What was the total amount spent on pie by the Math Club?

(A) \$152   (B) \$144   (C) \$138   (D) \$143   (E) None of the answers (A) through (D) is correct.

20. The perimeter of a rectangular volleyball court is 180 feet. The court's width, w, is half its length, l. Which system of linear equations could be used to determine the dimensions, in feet, of the volleyball court?

(A) $l + w = 180$ and $w = \frac{1}{2}l$ (B) $2l + 2w = 180$ and $w = \frac{1}{2}l$ (C) $l + w = 180$ and $l = \frac{1}{2}w$
(D) $2l + 2w = 180$ and $l = \frac{1}{2}w$ (E) None of the answers (A) through (D) is correct.

21. Six high school marching bands will perform in Class AAA at this year's Battle of the Bands competition. The order of performances is determined at random. What is the probability that the bands will perform in alphabetical order by school name?

(A) 21/470   (B) 1/720   (C) 6/21   (D) 1/6   (E) 1/120

22. If a and b are positive integers where $a + b < 20$ and $a > 7$, what is the greatest possible value of b?

(A) 11   (B) 12   (C) 13   (D) 20   (E) None of the answers (A) through (D) is correct.

23. Solve for c: $a(b + c) + b(a + c) = d$.

(A) $\frac{d}{ab} - 2$ (B) $\frac{a+b}{d-2ab}$ (C) $\frac{d-2ab}{2}$ (D) $\frac{d-2ab}{a+b}$ (E) None of the answers (A) through (D) is correct.

24. For how many pairs of positive integers (x, y) is $-4x - 2y = -100$?

(A) 25   (B) 50   (C) 18   (D) 24   (E) 27

25. Let I be the number of ordered pair solutions consisting only of integers to the system of inequalities $y \geq x - 1$ and $x - y \geq 0$. Find I.

(A) $I = 0$ (B) $I = 1$ (C) $1 < I \leq 10$ (D) $10 < I \leq 100$ (E) $I > 100$

26. One-half of the smaller of the two consecutive even integers is equal to two more than the larger. What is their product?

(A) 32   (B) 178   (C) 24   (D) 80   (E) None of the answers (A) through (D) is correct.

27. Four people play a game called Odd Man Out where each of the four people toss a fair quarter and there is an "odd man out" if three people toss heads and one person tosses tails or if three people toss tails and one person tosses heads. Each person throws a quarter once; what is the probability that there is an "odd man out"?

(A) 1/5   (B) 1/4   (C) 2/5   (D) 1/2   (E) 3/4

28. One of Thom and Tom always lies on Tuesdays, Wednesdays and Thursdays, and always tells the truth on the other days of the week. The other always lies on Fridays, Saturdays and Sundays, and always tells the truth the other days of the week. At noon, the two had the following conversation:
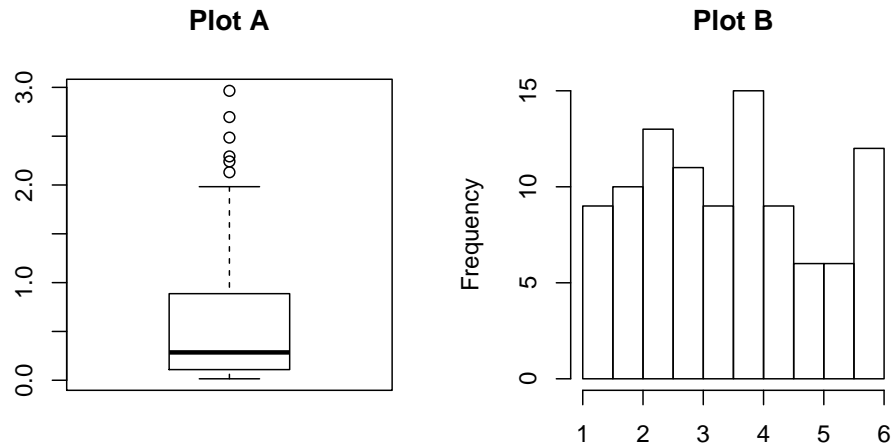
> **Thom:** I lie on Sundays.
>
> **Tom:** I will lie tomorrow.
>
> **Thom:** I lie on Mondays.

This conversation takes place on a

(A) Monday   (B) Tuesday   (C) Wednesday   (D) Thursday   (E) Friday

29. Plots A and B below represent two different data sets each consisting of 100 observations.
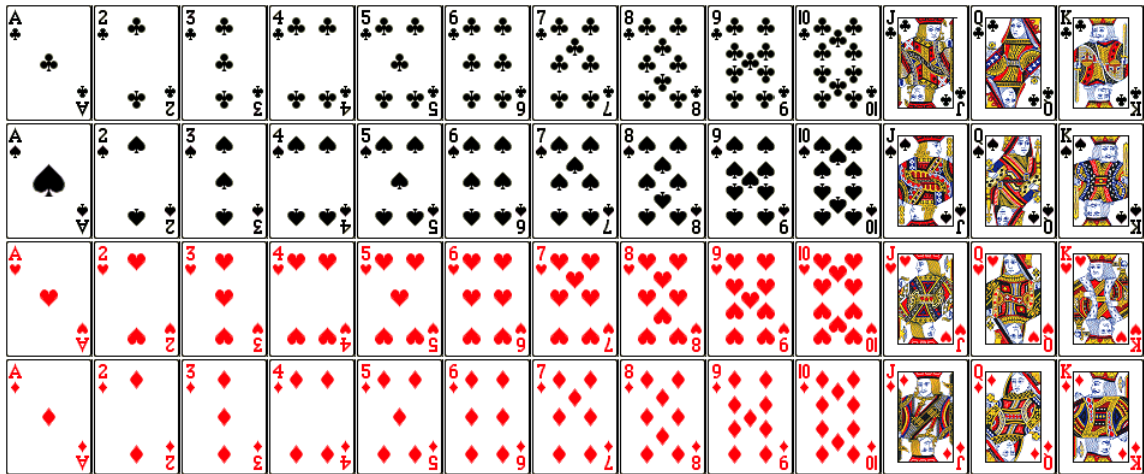

Consider the statements below:

- I: 75% of the observations in Plot A are smaller than all of the observations in Plot B.
- II: The mean of Plot A is greater than the median of Plot A.
- III: The range of Plot A is less than the range of Plot B.

(A) Only statement I is correct.   (B) Only statement II is correct.
(C) Only statement III is correct.   (D) Exactly two of I, II and III are true.
(E) All three statements I, II and III are true.

30. A deck of playing cards contains 52 cards, of which 26 are black and 26 are red. The 26 black cards are divided into two suits of 13 spades (♠) and 13 clubs (♣). The 26 red cards are divided into two suits of 13 hearts (♡) and 13 diamonds (♢). Each of the suits has cards of ranks A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q and K. The image below shows each of the cards in a standard deck – the first two rows of cards are black and the last two rows of cards are red. In the game of Blackjack, two cards are dealt to a person, and the best possible hand after the first two cards are dealt is a "Blackjack" where an A is dealt as one of the cards and the other card is 10, J, Q or K. The cards in a "Blackjack" do not need to be in the same suit, so, for example, an Ace of diamonds ($A \diamondsuit$) and 10 of spades (10 ♠) form a "Blackjack", as do ($A \clubsuit$) and ($K \clubsuit$). How many two-card combinations will form a "Blackjack"?

(A) 16 (B) 20 (C) 24 (D) 48 (E) 64