

HIGH SCHOOL MATHEMATICS CONTEST
Sponsored by
THE MATHEMATICS DEPARTMENT
of
WESTERN CAROLINA UNIVERSITY

LEVEL II TEST
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DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each of the 30 problems there are listed up to 5 possible answers. You are to work each problem and determine which is the correct answer. Indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. Here is a sample question and answer:

1. If $2x = 3$, then x equals:

- (A) $\frac{2}{3}$ (B) 3 (C) 6 (D) $\frac{3}{2}$ (E) None of the answers (A) through (D) is correct.

The correct answer for the sample is " $\frac{3}{2}$," which is answer (D); therefore, you should answer this question by making a heavy black mark under space D as indicated below.

A B C D E

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any question. If you are unable to work any particular problem, it is to your advantage to guess at the answer rather than leave it blank. Make no stray marks of any kind on your answer sheet.

When told to do so, open your test booklet to page 2 and begin work. When you have finished one page, go on to the next page. The working time for the entire test is 70 minutes.

The use of calculators is not permitted.

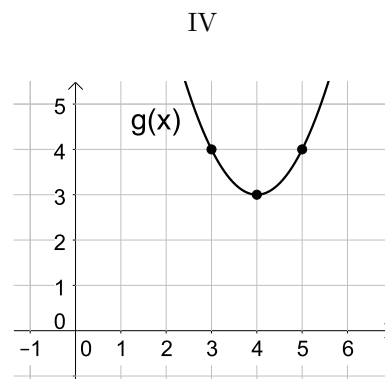
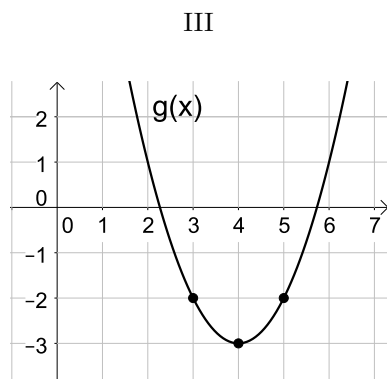
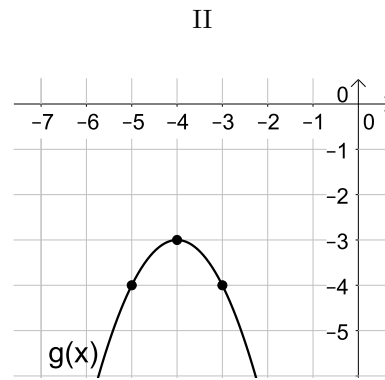
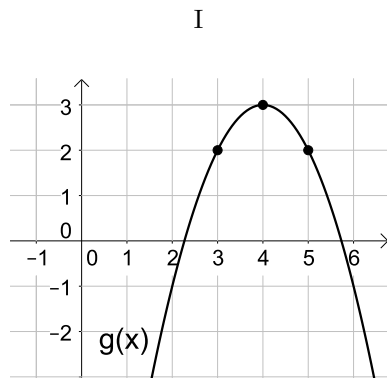
1. Simplify the statement $[(5 \times 2 - 12 \div 2)^2 - 8 \div 4] \div [-2 + (3 \times 4 - 14)^2]$
 (A) $\frac{-1}{2}$ (B) 7 (C) 12 (D) $\frac{-7}{8}$ (E) None of the answers (A) through (D) is correct.

2. 100 Freshmen were asked to select either statistics or calculus as a math course to take next semester. These students were also asked to select chemistry or astronomy as a science course to take next semester. Below is a contingency table that represents the selections these students made:

	Statistics	Calculus
Chemistry	26	15
Astronomy	42	17

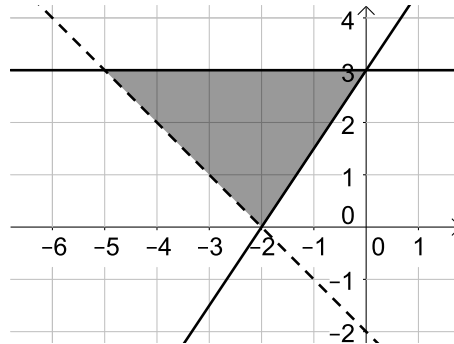
Using the table above, what is the probability that a student chose to take calculus or the student chose to take astronomy?

- (A) $\frac{17}{20}$ (B) $\frac{29}{50}$ (C) $\frac{37}{50}$ (D) $\frac{17}{100}$ (E) None of the answers (A) through (D) is correct.
3. Suppose $f(x) = x^2$. Which graph represents $g(x) = -f(x - 4) + 3$, a transformation of the function $f(x)$?



- (A) graph I (B) graph II (C) graph III (D) graph IV
 (E) None of the answers (A) through (D) is correct.

4. Which inequalities, listed below the graph, are true about the system of equations that would represent the graph shown below?



- I: $y \leq 3$
 II: $y \geq \frac{3}{2}x + 3$
 III: $y \geq -x - 2$

- (A) Only I is true (B) Only III is true (C) Both II and III are true (D) I, II, and III are true
 (E) None of the answers (A) through (D) is correct.

5. Find the equation of the line passing through the point $(8, 2)$ and perpendicular to $4x + 3y = 9$.

- (A) $y = \frac{3}{4}x + 2$ (B) $y = \frac{-3}{4}x + 3$ (C) $y = \frac{4}{3}x + 5$ (D) $y = \frac{-4}{3}x + 1$
 (E) None of the answers (A) through (D) is correct.

6. Given the equation of a circle, $x^2 + 6x + y^2 - 8y = 11$, find the distance between the center of the circle and the origin.

- (A) 25 (B) $\sqrt{5}$ (C) 6 (D) 5 (E) None of the answers (A) through (D) is correct.

7. Which of the following tables represent two events, A and B , that are independent?

I

	A	$\sim A$
B	17	41
$\sim B$	16	26

II

	A	$\sim A$
B	15	45
$\sim B$	10	30

III

	A	$\sim A$
B	20	37
$\sim B$	33	10

- (A) I only (B) II only (C) III only (D) Both I and III
 (E) None of the answers (A) through (D) is correct.

8. Which of the following functions has a graph with no vertical asymptote?

$$f(x) = \overset{\text{I}}{\frac{x^2 + 3x + 2}{x^2 + 1}} \qquad f(x) = \overset{\text{II}}{\frac{x^2 + 8x + 15}{x + 3}} \qquad f(x) = \overset{\text{III}}{\frac{x^2 + 5x + 6}{x - 3}}$$

(A) I only (B) II only (C) III only (D) Both I and II (E) None of the answers (A) through (D) is correct.

9. Suppose an Easter basket contains 2 peeps, 5 candy eggs, and 2 mini peanut butter cups. Jacob randomly picks one piece of candy from the basket and eats it. Wilhelm then randomly picks one piece of the remaining candy and eats it. What is the probability that at least one of them ate a candy egg?

(A) $\frac{5}{6}$ (B) $\frac{5}{9}$ (C) $\frac{4}{9}$ (D) $\frac{1}{6}$ (E) None of the answers (A) through (D) is correct.

10. Find the equation of the line that connects the vertex of the parabola $y = x^2 - 4x + 8$ with the center of the circle $(x - 5)^2 + (y + 3)^2 = 4$.

(A) $y = \frac{4}{3}x + \frac{32}{3}$ (B) $y = -\frac{1}{2}x - \frac{13}{2}$ (C) $y = -\frac{7}{3}x + \frac{26}{3}$ (D) $y = \frac{5}{2}x + \frac{23}{4}$
(E) None of the answers (A) through (D) is correct.

11. What is the value of x if $3(9^{2x+1}) = \frac{27^{x-8}}{3^{3-5x}}$?

(A) $\frac{29}{4}$ (B) -4 (C) $\frac{15}{2}$ (D) $\frac{-29}{6}$ (E) None of the answers (A) through (D) is correct.

12. Which of the following is equivalent to $\frac{4x^3y^2z^5}{6xy^{7/3}z^4} \div \frac{3xy^{3/2}z^2}{12xy^5z^3}$.

(A) $\frac{8}{3}x^2y^{19/6}z^2$ (B) $\frac{3}{2}xy^{13/6}z^2$ (C) $\frac{1}{6}x^3y^{7/6}z^3$ (D) $\frac{5}{12}x^3y^{11/6}z^3$
(E) None of the answers (A) through (D) is correct.

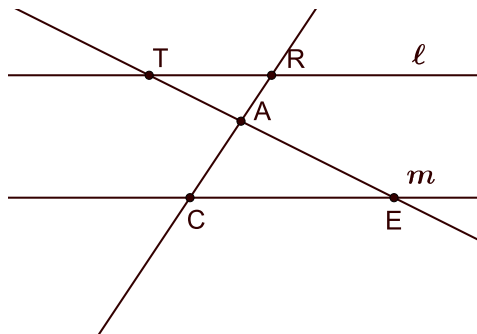
13. Which of the following solutions is equivalent to $(4 + 3i)^2 - \left(\frac{2}{1 - i}\right)$

(A) $24 + 23i$ (B) $6 + 11i$ (C) $5 + 26i$ (D) $6 + 23i$
(E) None of the answers (A) through (D) is correct.

14. Solve the equation $\log_4(x + 3) = 16$.

(A) 1 (B) 2 (C) -1 (D) 9 (E) None of the answers (A) through (D) is correct.

15. In the figure shown below, lines ℓ and m are parallel. If the length of $\overline{TA} = 3$, the length of $\overline{TE} = 12$, and the length of $\overline{RA} = 2$, what is the length of \overline{AC} ?



- (A) 4 (B) 6 (C) 8 (D) 10 (E) None of the answers (A) through (D) is correct.

16. The right triangle ABC has sides of the following lengths:

$$AB=24, \quad BC=7, \quad AC=25$$

Let there exist a point D so that D is the midpoint of AB. What is the length of CD?

- (A) $\sqrt{139}$ (B) $\sqrt{193}$ (C) 12 (D) 16 (E) None of the answers (A) through (D) is correct.
17. The equation $\frac{x^2}{4} - \frac{y^2}{36} = 2$ has a graph that is symmetric with respect to the:
- (A) x -axis (B) y -axis (C) x -axis and y -axis (D) x -axis, y -axis, and the origin (E) None of the answers (A) through (D) is correct.

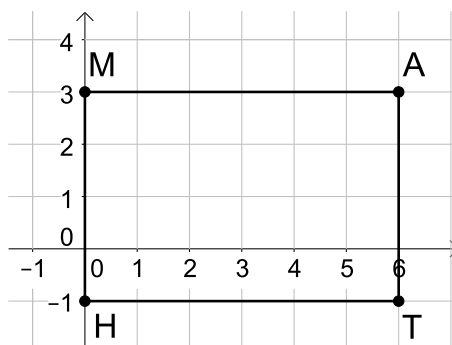
18. For what value of k will the system of equations

$$\begin{aligned} 2x - 3y &= -4 \\ -3x + ky &= 6 \end{aligned}$$

have an infinite number of solutions?

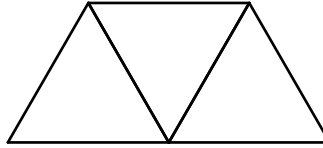
- (A) $9/2$ (B) 2 (C) $4/3$ (D) 1 (E) None of the answers (A) through (D) is correct.
19. A fifth-degree polynomial function with real coefficients has five zeros. It is known that three of them are 3, i , and $1 + i$. What is the product of all five zeros?
- (A) i (B) 0 (C) 6 (D) -6 (E) None of the answers (A) through (D) is correct.

20. Which of the following statements is true?
- (A) A second-degree polynomial with real coefficients has two real roots (not necessarily distinct).
 (B) A function cannot have more than one horizontal asymptote.
 (C) Doubling the radius of circle will double its circumference.
 (D) If the domain of a function $f(x)$ is \mathbb{R} , then it has an inverse $f^{-1}(x)$ with domain \mathbb{R} .
 (E) None of the answers (A) through (D) are true.
21. The rectangle shown below has vertices M at (0,3), A at (6,3), T at (6,-1), and H at (0,-1). If the center of dilation is the origin, dilate the rectangle using a scale factor of $\frac{1}{2}$, then rotate the resulting figure counterclockwise by 90° . What is the sum of the y -coordinates of vertices A and T after applying the transformation described here?



- (A) 2 (B) 3 (C) -1 (D) 1 (E) None of the answers (A) through (D) is correct.
22. Mike has 10 coins in his pocket. Some are quarters, some are dimes, and the rest are nickels. Aaron has twice as many quarters as Mike and half as many dimes, but the same number of nickels. Scott has the same number of quarters and dimes as Mike, but no nickels. If Aaron has \$2.75, and Scott has \$1.45, how much money does Mike have in his pocket?
- (A) \$1.60 (B) \$1.85 (C) \$1.35 (D) \$1.50
 (E) None of the answers (A) through (D) is correct.
23. If square $ABCD$ has the same area as a circle of radius 4, what is the length of line segment AC ?
- (A) 8π (B) $4\sqrt{2\pi}$ (C) $8\sqrt{\pi}$ (D) $\sqrt{8\pi}$ (E) None of the answers (A) through (D) is correct.
24. Which of the following is equivalent to the statement: $\log\left(\frac{(x+2)^3}{x^5(2x-3)^7}\right)$?
- (A) $-2\log(x) + 3\log(2) - 7\log(2x) + 7\log(3)$
 (B) $-2\log(x) + 3\log(2) + 7\log(2x) - 7\log(3)$
 (C) $3\log(x+2) - 5\log(x) - 7\log(2x-3)$
 (D) $3\log(x+2) - 5\log(x) + 7\log(2x-3)$
 (E) None of the answers (A) through (D) is correct.

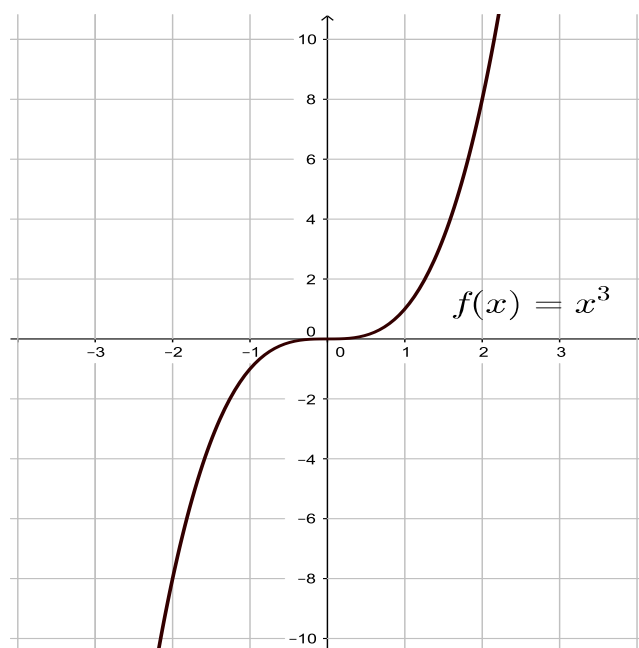
25. Suppose 3 equilateral triangles are placed as shown in the image below:



If the outer perimeter of the object shown above is 40 feet, what is the area of the object?

- (A) $16\sqrt{3}$ ft² (B) $48\sqrt{3}$ ft² (C) $32\sqrt{5}$ ft² (D) $96\sqrt{5}$ ft²
(E) None of the answers (A) through (D) is correct.
26. Suppose $f(x) = -5x + 2$ and $g(x) = 50x^2 - 6$. What is the value of $(f^{-1} \circ g)(x)$?
- (A) $-10x^2 + 4/5$ (B) $10x^2 - 8/5$ (C) $\frac{1}{-250x^2 + 32}$ (D) $-10x^2 + 8/5$
(E) None of the answers (A) through (D) is correct.
27. Suppose A varies jointly with $\frac{1}{r-3}$ and t^3 . When $r = 6$ and $t = 3$, $A = 18$. What is A if $r = 11$ and $t = 4$?
- (A) 16 (B) 72 (C) $\frac{16}{9}$ (D) 36 (E) None of the answers (A) through (D) is correct.
28. Two circles of different radii are drawn such that they have the same center point. A chord of the larger circle is drawn tangent to the smaller circle and measures 20 inches. What is the area bounded between the two circles?
- (A) 25π in² (B) 50π in² (C) 100π in² (D) 200π in²
(E) None of the answers (A) through (D) is correct.

29. The graph of $f(x) = x^3$ is illustrated below:



If $g(x) = f(x + 1) + 2$, then what is $g^{-1}(10)$?

(A) 0 (B) 1 (C) 2 (D) 3 (E) None of the answers (A) through (D) are true.

30. If $a + b = \frac{1}{2}$ and $a^2 + b^2 = 1$, then what does $a^3 + b^3$ equal?

(A) 7 (B) $\frac{1}{8}$ (C) $-\frac{3}{8}$ (D) $\frac{11}{16}$ (E) None of the answers (A) through (D) is correct.