HIGH SCHOOL MATHEMATICS CONTEST
Sponsored by
THE MATHEMATICS DEPARTMENT
of
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LEVEL I TEST
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DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each of the 30 problems there are listed up to 5 possible answers. You are to work each problem and determine which is the correct answer. Indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. Here is a sample question and answer:

1. If $2x = 3$, then x equals:

   (A) $\frac{2}{3}$ (B) 3   (C) 6   (D) $\frac{3}{2}$ (E) None of the answers (A) through (D) is correct.

The correct answer for the sample is "$\frac{3}{2}$," which is answer (D); therefore, you should answer this question by making a heavy black mark under space D as indicated below.

A   B   C   D   E

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any question. If you are unable to work any particular problem, it is to your advantage to guess at the answer rather than leave it blank. Make no stray marks of any kind on your answer sheet.

When told to do so, open your test booklet to page 2 and begin work. When you have finished one page, go on to the next page. The working time for the entire test is 70 minutes.

The use of calculators is not permitted.

1. The sum of three consecutive odd integers is 93. What is the smallest of the three numbers?

   (A) 29   (B) 31   (C) 33   (D) 35   (E) None of the answers (A) through (D) is correct.

2. Suppose the triangle $\triangle ABC$ shown in the diagram is an equilateral triangle. If the line BC is described by an equation of the form $y = mx + b$ for some m and b, then find m.
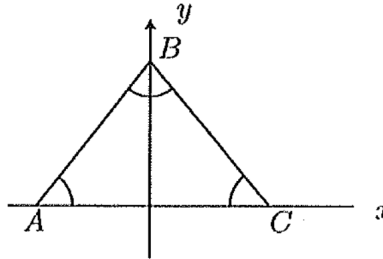
   (A) 60   (B) 90   (C) -1/2   (D) $-\sqrt{3}$ (E) -60

3. In a group of 120 people, 90 have an age of more than 30 years, and the others have an age of less than 20 years. If a person is selected at random from this group, what is the probability the person's age is less than 20?

   (A) $\dfrac{1}{120}$ (B) $\dfrac{1}{90}$ (C) $\dfrac{1}{20}$ (D) $\dfrac{1}{30}$ (E) $\dfrac{1}{4}$

4. What is the sum of the solutions of $x^2 - 5x + 7 = 0$?

   (A) -7   (B) -5   (C) 0   (D) 5   (E) 7

5. In a small town of 1000 people, all of the residents have an annual income close to $30,000. One day, a person with an annual income of $4,000,000 moves into the town. Which of the following statements best represents the resulting change in the town's income distribution?

   (A) The median and mean will both stay close to the same   (B) The median will increase significantly, but the mean will not   (C) The mean will increase significantly, but the median will not   (D) Both the mean and the median will increase significantly   (E) None of the answers (A) through (D) is correct

6. Suppose L is a line segment with one endpoint at $(3, 11)$. If the midpoint of L is at $(7, 14)$, how long is L?

   (A) 5   (B) 7   (C) 10   (D) 14   (E) None of the answers (A) through (D) is correct

7. Let n be the largest positive integer such that 2^n divides the product of the first 12 positive integers. What is the value of n?

   (A) 5   (B) 10   (C) 12   (D) 20   (E) None of the answers (A) through (D) is correct

8. For the data set
$$2, 3, 3, 4, 5, 6, 9, 9, 9, 9, 10$$
which of the following quantities is the smallest?

(A) The median   (B) The mode   (C) The mean   (D) The range
(E) More than one quantity has the same smallest value.

9. What is the y-intercept of the line that passes through the point $(1, 3)$ and is parallel to the line that passes through the points $(2, 7)$ and $(3, 29)$?

(A) -19   (B) 19   (C) -22   (D) 22   (E) None of the answers (A) through (D) is correct

10. In a class of 10 boys and 18 girls, the boys' average score on the final exam was 85 and the girls' average score was 90. Rounded to the nearest integer, what was the average score of the whole class?

(A) 83   (B) 85   (C) 88   (D) 89   (E) 90

11. If the points $(3, 6), (6, 11)$ and $(8, c)$ are collinear, what is the value of c?

(A) $12\frac{1}{3}$ (B) 13   (C) 14   (D) $14\frac{1}{3}$.   (E) $14\frac{2}{3}$

12. The volume of a cylinder is given by $V = \pi r^2 h$, where r is the radius of the base of the cylinder and h is the height of the cylinder. If the radius of the base of a cylinder is doubled and its height is tripled, by what number is the volume multiplied?

(A) 2   (B) 3   (C) 6   (D) 9   (E) 12

13. In the diagram below $\overline{BE}$ and $\overline{CD}$ are parallel. If $\angle CAD$ measures $60°$, $\angle ABE$ measures $2x$ degrees, and $\angle CDE$ measures $x + 30$ degrees, then find x.
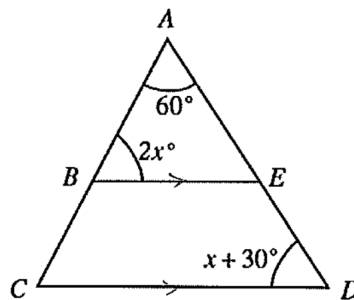
(A) 15   (B) 30   (C) 45   (D) 60   (E) None of the choices (A) through (D) is correct

14. Each year for three consecutive years, an unlucky investor loses 20% of his money. Which of the following describes how much money the investor has left at the end of the third year, as a percentage of the original amount?

(A) More than 80% of the original amount   (B) More than 50% and less than 80% of the original amount   (C) More than 20% and less than 50% of the original amount   (D) Less than 20% of the original amount   (E) None of the choices (A) through (D) is correct

15. If $f(x) = \sqrt{2x-5}$, then for what value of x does $f(x-1) = 7$?

(A) 28   (B) 21   (C) 49   (D) 26   (E) 25

16. If $x + y = 0$ and $x \neq 0$ then what is the value of $\dfrac{x^{1931}}{y^{1931}}$

(A) 1   (B) -1   (C) 2   (D) -2   (E) None of the choices (A) through (D) is correct

17. Find the sum of the factors of $3x^2 + 2x - 5$.

(A) $4x + 4$ (B) $4x - 4$ (C) $4x - 6$ (D) $4x + 6$ (E) $4x - 5$

18. A biologist has collected the following data on the relationship between temperature (measured in F°) and the chirps per minute of a certain species of cricket (in chirps per minute).

Temperature	50	55	60	65	70
Chirping rate	20	46	80	92	113

A linear model for chirping rate as a function of temperature is given by $\hat{y} = 4.6x - 207$. What does this model predict the chirping rate will be if the temperature is $60F^\circ$? Round your answer to the nearest integer.

(A) 54   (B) 68   (C) 69   (D) 79   (E) 80

19. A small circle just fits inside a semicircle. What is the ratio of the area of the small circle to the area of the semicircle containing it?
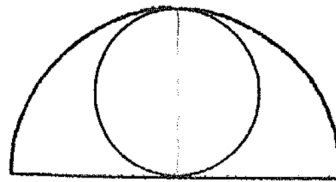
(A) 1:1   (B) 1:2   (C) 1:3   (D) 2:3   (E) 3:4

20. If $f(x)$ is a quadratic polynomial with $f(1) = 0$, $f(2) = -4$ and $f(3) = 0$, what is $f(4)$?

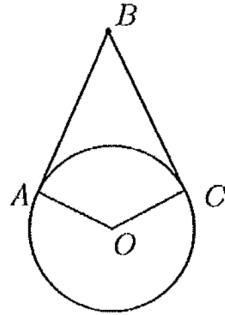(A) 2   (B) 4   (C) 6   (D) 8   (E) None of the choices (A) through (D) is correct

21. If $\dfrac{x}{y^2} = 7$ and $\dfrac{y^4}{z} = 3$, what is the value of $\dfrac{x^2}{z}$?

(A) $\frac{7}{9}$ (B) $\frac{9}{7}$ (C) 21   (D) 63   (E) None of the choices (A) through (D) is correct

22. By coincidence, my father and I have the same birthday: March 15. On March 15, 2018, I will be the same age as my father was on March 15, 1989, and my father will be 65 years old. How old will I be on this date?

(A) 29   (B) 30   (C) 33   (D) 36   (E) None of the choices (A) through (D) is correct

23. Suppose that $w^2 - 2w = 0$ and $2y^2 - y = 0$, then $\dfrac{w}{y}$ is equal to

(A) $\dfrac{w^2}{4y^2}$ (B) $\dfrac{4w^2}{y^2}$ (C) $\dfrac{w^2}{y^2}$ (D) $\dfrac{2w^2}{y^2}$ (E) $\dfrac{w^2}{2y^2}$

24. If $x\#y$ is defined to be $x^2 + y$, what is the value of $(1\#(2\#(3\#4)))$?

(A) 10   (B) 16   (C) 20   (D) 100   (E) None of the choices (A) through (D) is correct

25. If $\dfrac{a}{b} = \dfrac{9}{4}$ and $\dfrac{b}{c} = \dfrac{5}{3}$, then $\dfrac{a-b}{b-c}$ is equal to:

(A) $\dfrac{4}{1}$ (B) $\dfrac{25}{8}$ (C) $\dfrac{7}{12}$ (D) $\dfrac{5}{2}$ (E) There is not enough information to determine this answer

26. The expression

$$\frac{9^4 \left(3^{2x+1}\right)^3}{81^5}$$

can be simplified to have the form 27^u. What is the value of u?

(A) $2x - 3$ (B) $2x - 1$ (C) $2x + 1$ (D) $2x + 3$ (E) None of the choices (A) through (D) is correct

27. Adam can mow the lawn in 4 hours and his son can mow the same amount in 12 hours. How long will it take them to mow the lawn if they work together?

(A) 2 hours   (B) 2.5 hours   (C) 3 hours   (D) 3.5 hours   (E) None of the other choices is correct

28. If $f(x) = 5 - 3x$, calculate the value of $f^{-1}(2)$.

(A) -1   (B) 1   (C) 3   (D) -3   (E) 0

29. Solve the inequality
$$|x + 1| < |x^2 + 2x + 2|$$

(A) $(-\infty, +\infty)$ (B) $(-1, +\infty)$ (C) $(-\infty, -1)$ (D) $(-1, 1)$ (E) There are no solutions

30. In the diagram below, O is the center of the circle, and $\overline{AB}$ and $\overline{BC}$ are tangent lines to the circle with points of tangency A and C respectively. If $\angle O$ measures $90°$ then the measure of $\angle B$ is

(A) $45°$ (B) $60°$ (C) $90°$ (D) $120°$ (E) None of the other choices (A) through (D) is correct