

HIGH SCHOOL MATHEMATICS CONTEST  
Sponsored by  
THE MATHEMATICS DEPARTMENT  
of  
WESTERN CAROLINA UNIVERSITY

LEVEL II TEST  
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DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each of the 30 problems there are listed up to 5 possible answers. You are to work each problem and determine which is the correct answer. Indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. Here is a sample question and answer:

1. If  $2x = 3$ , then  $x$  equals:

- (A)  $\frac{2}{3}$  (B) 3 (C) 6 (D)  $\frac{3}{2}$  (E) None of the answers (A) through (D) is correct.

The correct answer for the sample is " $\frac{3}{2}$ ," which is answer (D); therefore, you should answer this question by making a heavy black mark under space D as indicated below.

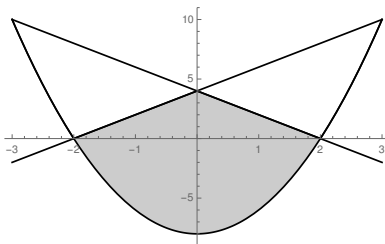
A B C D E

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any question. If you are unable to work any particular problem, it is to your advantage to guess at the answer rather than leave it blank. Make no stray marks of any kind on your answer sheet.

When told to do so, open your test booklet to page 2 and begin work. When you have finished one page, go on to the next page. The working time for the entire test is 70 minutes.

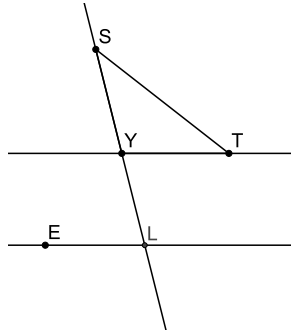
**The use of calculators is not permitted.**

- Suppose  $y$  varies inversely with the square of  $x$ . If  $x = 4$ , then  $y = \frac{1}{4}$ . What is  $y$  if  $x = 3$ ?  
 (A)  $\frac{1}{3}$  (B)  $\frac{1}{2\sqrt{3}}$  (C)  $\frac{4}{9}$  (D)  $\frac{4}{3}$  (E) None of the answers (A) through (D) is correct.
- Suppose peanuts cost \$2.50 per pound and walnuts cost \$4.50 per pound. You decide to make a mix of peanuts and walnuts so that you have 50 pounds of mixed nuts to sell at \$3.00 per pound. How many pounds of walnuts are in the mixture?  
 (A) 12.5 lbs (B) 31.25 lbs (C) 37.5 lbs (D) 18.75 lbs (E) None of the answers (A) through (D) is correct.
- Given the equation of a circle,  $(x - 2)^2 + (y + 3)^2 = 25$  and a quadratic equation,  $y = 3(x + 5)^2 - 4$ , find the distance between the center of the circle and the vertex of the quadratic equation.  
 (A)  $\sqrt{50}$  (B)  $\sqrt{58}$  (C)  $\sqrt{10}$  (D)  $\sqrt{98}$  (E) None of the answers (A) through (D) is correct.
- Mary is taking 5 exams and wants to earn an average score of 80%. Her scores on the first three exams were 92%, 75%, and 87%. What average score does she need on the last two exams to meet her goal?  
 (A) 73% (B) 66% (C) 87% (D) 84.66% (E) None of the answers (A) through (D) is correct.
- Which of the following combination of inequalities best describes the shaded region graphed below

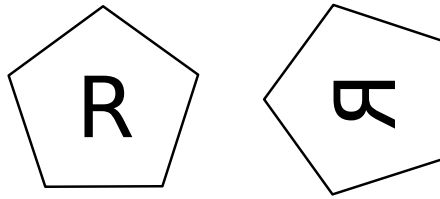


- $2x^2 - 8 \leq 4 - 2x$  and  $2x^2 - 8 \leq 4 + 2x$
  - either  $2x^2 - 8 \leq 4 - 2x$  or  $2x^2 - 8 \leq 4 + 2x$
  - either  $2x^2 - 8 \leq 4 - 2x$  or  $2x^2 - 8 \leq 4 + 2x$  but not both
  - either answer (B) or answer (C)
  - None of the answers (A) through (D) is correct.
- Find the area of a triangle which has vertices at  $(-4, 3)$ ,  $(-4, -2)$ , and  $(1, -2)$ .  
 (A)  $25\sqrt{2}$  (B) 25 (C)  $12.5\sqrt{2}$  (D)  $\frac{\sqrt{26}}{2}$  (E) None of the answers (A) through (D) is correct.
- What is the value of  $x$  if  $8^{-3x-2} = \frac{16^{2x+5}}{2^{3-2x}}$ ?  
 (A)  $\frac{29}{15}$  (B)  $\frac{-21}{16}$  (C)  $\frac{-11}{15}$  (D)  $\frac{-8}{19}$  (E) None of the answers (A) through (D) is correct.

8. In the figure shown below, lines  $\overline{YT}$  and  $\overline{EL}$  are parallel. Suppose triangle  $STY$  is an isosceles triangle such that  $SY = YT$ . If  $\angle YTS$  is  $37^\circ$  what is the size of  $\angle ELY$ ?



- (A)  $74^\circ$  (B)  $106^\circ$  (C)  $34^\circ$  (D)  $146^\circ$  (E) None of the answers (A) through (D) is correct.
9. A glass desk ornament is a hollow rectangular prism with both length and width equal to 5cm and height 8cm. The prism is filled with water so that when it is standing on its square base the depth of the water is 6cm. How deep is the water when the ornament is placed on a non-square face?
- (A) 3cm (B) 4cm (C)  $\frac{25}{8}$ cm (D)  $\frac{15}{4}$ cm (E) None of the answers (A) through (D) is correct.
10. The shape below on the left has been transformed to the shape on the right by a sequence of two geometric transformations.



If the origin is placed at the center of the figure then the transformations are

- (A) a  $90^\circ$  clockwise rotation followed by a flip over the  $x$ -axis.  
 (B) a flip over the  $y$ -axis followed by a flip over the  $x$ -axis.  
 (C) a  $90^\circ$  clockwise rotation followed by a flip over the  $y$ -axis.  
 (D) a  $90^\circ$  counter clockwise rotation followed by a flip over the  $y$ -axis.  
 (E) None of the answers (A) through (D) is correct.
11. Which of the following tables represent two events that are not independent?

I

	A	not A
B	32	17
not B	7	44

II

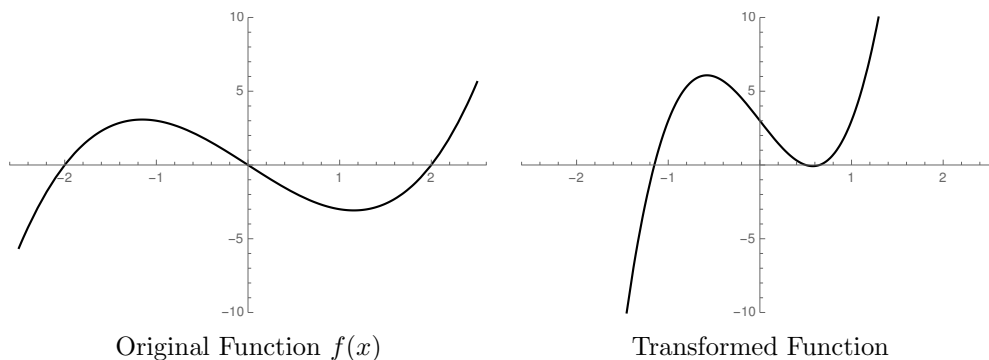
	A	not A
B	11	35
not B	51	3

III

	A	not A
B	9	27
not B	16	48

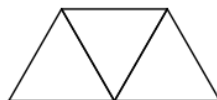
- (A) I only (B) II only (C) III only (D) Both I and II  
 (E) None of the answers (A) through (D) is correct.

12. The function graphed in the right hand figure below has been generated by applying some combination of shifts and transforms to the figure on the left.



If the original function is  $f(x)$  then the transformed function is

- (A)  $f(x/2) - 3$  (B)  $f(2x) + 3$  (C)  $2f(x - 3)$  (D)  $f(x + 3)/2$   
 (E) None of the answers (A) through (D) is correct.
13. Suppose Mary and Joe both work at a grocery store. On Saturday, Mary worked from 9AM to 3PM and Joe worked from 10AM to 7PM. Together Mary and Joe earned \$102. If Mary earns \$2 an hour more than Joe, then the average of their two wages is
- (A) \$9 per hour (B) \$6 per hour (C) \$8 per hour (D) \$7 per hour  
 (E) None of the answers (A) through (D) is correct.
14. Suppose 3 equilateral triangles are placed as shown in the image below:



If the area of the object shown above is  $3\sqrt{3}$  in<sup>2</sup>, what is the length of the side of one of the equilateral triangles?

- (A) 2 in (B) 1 in (C)  $\sqrt{3}$  in (D) 4 in  
 (E) None of the answers (A) through (D) is correct.
15. Suppose  $f(x) = \frac{6}{x+1}$  and  $g(x) = \frac{3}{x}$ . What is the value of  $f^{-1} \circ g(x)$ ?
- (A)  $18x - 1$  (B)  $\frac{2}{x} - 1$  (C)  $\frac{18}{x} + 1$  (D)  $2x - 1$   
 (E) None of the answers (A) through (D) is correct.

16. Mary, Frank, Susan and June have built houses in a popular video game. The four houses are arranged in a straight line from left to right and are each made from either wood, cobblestone, glass, or dirt. Each house is made from a different material and the following facts are known:

- The third house in the row is cobblestone.
- Mary's house is made from dirt.
- Frank's house is not at either end, but is further right of the glass house.
- Susan owns the fourth house.
- The first house is not made from dirt.

What is June's house made of?

- (A) Wood (B) Cobblestone (C) Glass (D) Dirt  
 (E) Not enough information to determine an answer.

17. Square  $ABCD$  has sides of length 10. Points  $E$ ,  $F$ ,  $G$ , and  $H$  are chosen on sides  $AB$ ,  $BC$ ,  $CD$ , and  $DA$ , respectively so that  $AE = BF = CG = DH = 2$ . Find the area of  $EFGH$ .

- (A) 50 (B) 100 (C) 64 (D) 68 (E) None of the answers (A) through (D) is correct.

18. If the product of two negative, consecutive, even integers is 168, find the sum of the integers.

- (A) 22 (B) -26 (C) 26 (D) -22 (E) None of the answers (A) through (D) is correct.

19. Which of the following solutions is equivalent to  $\frac{4}{2-i} + \frac{1}{3+i}$ , where  $i^2 = -1$

- (A)  $\frac{19-7i}{10}$  (B)  $\frac{73-29i}{24}$  (C)  $\frac{13-7i}{10}$  (D)  $\frac{73+29i}{24}$   
 (E) None of the answers (A) through (D) is correct.

20. A door is 4ft wide and 7ft high. If the door is standing open at a right angle with the door frame, what is the greatest distance (in feet) from the outer top corner of the door to a point on the door frame.

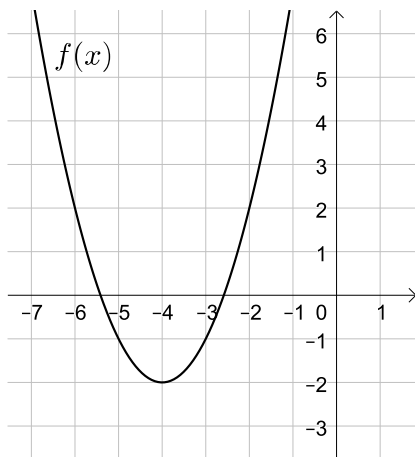
- (A)  $\sqrt{65}$  (B) 9 (C)  $\sqrt{97}$  (D) 15 (E) None of the answers (A) through (D) is correct.

21. Simplify the following fraction, assuming  $x$  and  $y$  are positive numbers

$$\frac{\left(\frac{x^{5/2}y}{\sqrt{xy}}\right)^3}{\frac{x^{-1}y^{-2}}{x\sqrt{y^3}}}$$

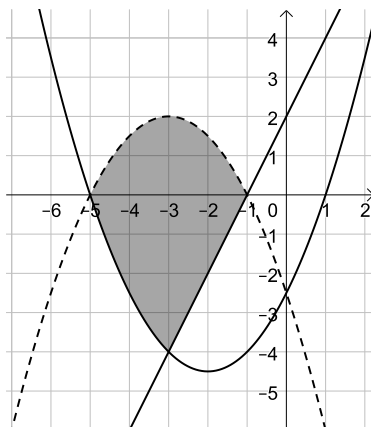
- (A)  $x^2y^3$  (B)  $x^8y^5$  (C)  $x^4y^4$  (D) 1 (E) None of the answers (A) through (D) is correct.

22. Suppose  $f(x) = x^2$ . The graph below represents a transformation of the function  $f(x)$ . Assume the graph has not been stretched vertically, or horizontally, but may have been shifted horizontally, or vertically.



Write an equation that represents the function  $f(x)$  after the transformations shown above.

- (A)  $f(x) = x^2 - 8x + 14$  (B)  $f(x) = x^2 + 8x + 16$  (C)  $f(x) = x^2 + 8x + 14$  (D)  $f(x) = x^2 - 8x + 16$   
 (E) None of the answers (A) through (D) is correct.
23. The solution to a system of inequalities is represented by the shaded region in the graph below. Based on this solution, which of the given inequalities are true?



I:  $y \leq -0.5(x + 3)^2 + 2$

II:  $y \geq 0.5(x + 2)^2 - 4.5$

III:  $y > 2x + 2$

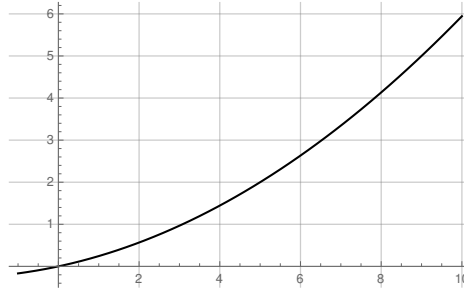
- (A) Only I is true (B) Only II is true (C) Both I and III are true (D) I, II, and III are true  
 (E) None of the answers (A) through (D) is correct.

24. Find all solutions to the equation

$$e^{2x} + 6e^x + 5 = 0$$

- (A)  $x = -1, -5$  (B)  $x = e, e^5$  (C)  $x = e^{-1}, e^{-5}$  (D)  $x = 0, \ln(5)$   
 (E) None of the answers (A) through (D) is correct.

25. The graph of  $f(x)$  is shown below



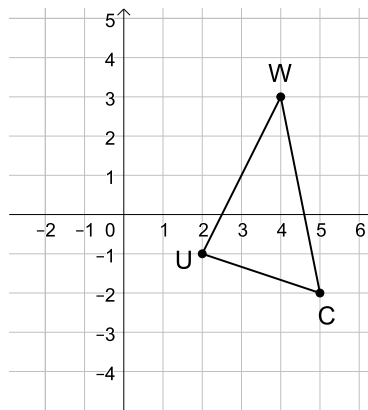
If  $f^{-1}(2x + 1) = 5$  then  $x$  is

- (A)  $x = \frac{1}{2}$  (B)  $x = 3$  (C)  $x = 2$  (D)  $x = \frac{1}{4}$   
 (E) None of the answers (A) through (D) is correct.

26. An equilateral triangle has side length 14. A point in the interior is the same distance to the midpoint of all three sides. What is the distance?

- (A)  $\frac{\sqrt{3}}{21}$  (B)  $\frac{7\sqrt{3}}{2}$  (C)  $\frac{7}{2}$  (D)  $\frac{7\sqrt{3}}{3}$  (E) None of the answers (A) through (D) is correct.

27. The triangle shown below has vertices W at (4,3), C at (5,-2), and U at (2,-1). If the center of transformation is the origin, dilate the triangle using a scale factor of  $\frac{1}{2}$ , then rotate the resulting figure counterclockwise by  $180^\circ$ . What is the sum of the  $y$ -coordinates of vertices W and C?

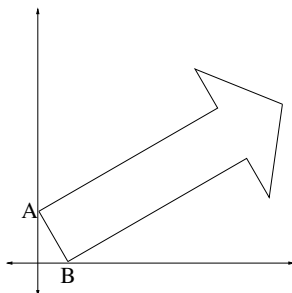


- (A) 0.5 (B) 1 (C) -0.5 (D) 0 (E) None of the answers (A) through (D) is correct.

28. Suppose the sum of two numbers  $x$  and  $y$  is equal to their product. Suppose also that  $3x + 3y = 16$ . What is  $|x - y|$ ?

- (A)  $\frac{8}{3}$  (B)  $\frac{4}{3}$  (C)  $\frac{1}{2}$  (D) 0 (E) None of the answers (A) through (D) is correct.

29. Consider the figure shown below



If point  $A$  is at  $(0, 6)$  and point  $B$  is at  $(2, 0)$  then the axis of symmetry of the arrow is

- (A)  $y = -3x + 6$  (B)  $y = \frac{1}{3}x + 6$  (C)  $y = -3x + \frac{7}{3}$  (D)  $y = \frac{1}{3}x + \frac{8}{3}$   
(E) None of the answers (A) through (D) is correct.

30. Which of the following describes the graph of the equation  $(x + y)^2 = x^2 + y^2$ ?

- (A) two lines (B) a single point (C) the empty set (D) a circle  
(E) None of the answers (A) through (D) is correct.